**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: -** Since the work beings 10 mins after the car is dropped, the time left to complete the work is 50 mins. Probability that service manager cannot meet his commitment = P(x>50) = 1-P(X<=50)

Where X is the time taken to complete work

Convert 50 to z-score

Standard normal variable Z = (X-*μ* )/*σ* =(X-45)/8

P(X<=50) = P(Z<(50-45)) =P(Z<=0.625) = 0.73237 = 73.237%

So, the probability that service manager will not meet his commitment is= 100-73.237= 0.2676

It is B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:-** Let X be the number of employees where = 38 and =6

1. Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X <= 44)

Z = (X-38)/6

Pr (X <=44) = Pr (Z <= (44 - 38)/6) = Pr(Z <=1)=84.13%

Probability for older than 44 = 100-84.13= 15.86%

Probability of age between 38 and 44 = 34.1345

So, the statement A is true

1. Probability of employees less than age of 30 = Pr(X<30)

Z= (30 - 38)/6

Pr(X <= 30) = Pr(Z <=(30 - 38)/6) = Pr(Z <= -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under

age 30 =0.0912\*400=36.48(or 36 employees).

Therefore, the statement B is also True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:-** X1 and X2 are two independent random variables.

2X1~ N (2 u,4 σ^2)

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

So, difference between them is

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:-**  a and b are symmetric about mean such that probability is 0.99

Probability of outside a and b is 1- 0.99= 0.01

The Probability towards left from a = -0.005

The Probability towards right from b = +0.005

Z= (X- μ) / σ

For Probability 0.005 the Z Value is -2.57.

Z \* σ + μ = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.4

Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:-** Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

So, company profits are

P~N(12,52)

1. Range containing 95% probability for profit is

= (12-196\*5.12+1.96\*5)

= ($2.2M, $ 1026M)

=(Rs99M,1026M)

1. 5th percentile is

=P(z<=(p-12/5))

= 0.05=(p-12)/5

= 1.644=p=12-8.22

=$3.78M

= 170M Rs.

1. Loss is when profit < 0

Thus: p < 0The first division of company, thus have larger probability of making a loss each year